LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **NOVEMBER 2023**

PMT1MC03 – ORDINARY DIFFERENTIAL EQUATIONS

Date: 06-11-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
1	**Answer the following**	
a)	State Lipschitz condition.	
b)	Define linear dependence solutions.	
c)	Describe the systems of first order equations.	
d)	Define regular singular point.	
e)	State Sturm's separation theorem.	

SECTION A – K2 (CO1)

	Answer ALL the questions	**(5 x 1 = 5)**
2	**Choose the correct answer**	
a)	The second approximate solution of $x' = tx$, $x(0) = 1$, as per Picard's successive approximation method (i) $1 + t$ (ii) $1 + \frac{t}{2}$ (iii) $1 + t^2$ (iv) $1 + \frac{t^2}{2}$	
b)	Which of the following is the solution of the equation $x'' + 4x = 0$? (i) $c_1e^{2t} + c_2e^{-2t}$ (ii) $c_1e^{2it} + c_2e^{-2it}$ (iii) $c_1cos2t + c_2sint$ (iv) none of these	
c)	Let Φ be a fundamental matrix for the system $x' = A(t)x$. Then $\Phi(t+s) =$ (i) $\Phi(t) + \Phi(s)$ (ii) $\Phi(t) - \Phi(s)$ (iii) $\Phi(t)\Phi(s)$ (iv) $\Phi(t)/\Phi(s)$	
d)	The Bessel function (p is an integer), $J_{-p}(t) =$ (i) $J_p(t)$ (ii) $-J_p(t)$ (iii) $-J_p(n)$ (iv) none of these	
e)	The equation $x'' + x = 0$ is (i) oscillatory (ii) non-oscillatory (iii) neither (i) nor (ii) (iv) both (i) and (ii)	

SECTION B – K3 (CO2)

	Answer any THREE of the following	**(3 x 10 = 30)**
3	Apply the method of variation of parameters to find the solution of $x' + a(t)x = b(t)$.	
4	Use Wronskian to classify the following sets of functions as linearly independent or dependent: (i) $sint, sin2t, sin3t$ on $I = [0,2\pi]$, (ii) $1, t, t^2, t^3$ on R.	
5	Show that $\Phi(t) = \begin{bmatrix} e^{-3t} & te^{-3t} & t^2e^{-3t}/2 \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$ is a fundamental matrix for a linear system $x' = A(t)x$	

	where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$.
6	For distinct values of n and m, prove that $\int_{-1}^{1} P_n(t)\, P_m(t)\, dt = 0$.
7	Let r_1, r_2 and p be continuous functions on (a, b) and $p > 0$. Assume that x and y are real solutions of $(px')' + r_1 x = 0$ and $(py')' + r_2 y = 0$ respectively on (a, b). If $r_2(t) \geq r_1(t)$ for $t \in (a, b)$, show that between any two consecutive zeros t_1, t_2 of x in (a, b), there exists at least one zero of y in $[t_1, t_2]$.
	SECTION C – K4 (CO3)
	Answer any TWO of the following (2 x 12.5 = 25)
8	Derive the various solutions of second order homogeneous differential equation with constant coefficients.
9	Solve the initial value problem $x'' - 2x' + x = 0, x(0) = 0, x'(0) = 1$ by converting into system of equations.
10	Obtain the integral representation of Bessel function.
11	Explain the Hille-Wintner comparison theorem.
	SECTION D – K5 (CO4)
	Answer any ONE of the following (1 x 15 = 15)
12	Let $x' = A(t)x$ be a linear system where $A: I \to M_n(R)$ is continuous. Suppose a matrix Φ satisfies the system, evaluate $(\det \Phi)'$ and discuss that if Φ is a fundamental matrix if and only if $\det \Phi \neq 0$.
13	Determine the solution of the Legendre equation $(1 - t^2)x'' - 2tx' + p(p + 1)x = 0$.
	SECTION E – K6 (CO5)
	Answer any ONE of the following (1 x 20 = 20)
14	Develop the conditions for the existence of a unique solution for the first order initial value problem $x' = f(t, x), x(t_0) = x_0$
15	Suppose that there are two types of living things that need the same type of food supply to survive. Create a mathematical model to explain this occurrence, and explain how the model could be used to predict the extinction of any particular species.

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